Figure 3.1  This odd-looking headgear provides the user with a virtual world. (credit: fil/Pixabay)

Chapter Outline

3.1 Graph Linear Equations in Two Variables
3.2 Slope of a Line
3.3 Find the Equation of a Line
3.4 Graph Linear Inequalities in Two Variables
3.5 Relations and Functions
3.6 Graphs of Functions

Introduction

Imagine visiting a faraway city or even outer space from the comfort of your living room. It could be possible using virtual reality. This technology creates realistic images that make you feel as if you are truly immersed in the scene and even enable you to interact with them. It is being developed for fun applications, such as video games, but also for architects to plan buildings, car companies to design prototypes, the military to train, and medical students to learn.

Developing virtual reality devices requires modeling the environment using graphs and mathematical relationships. In this chapter, you will graph different relationships and learn ways to describe and analyze graphs.

3.1 Graph Linear Equations in Two Variables

Learning Objectives

By the end of this section, you will be able to:

- Plot points in a rectangular coordinate system
- Graph a linear equation by plotting points
- Graph vertical and horizontal lines
- Find the x- and y-intercepts
- Graph a line using the intercepts

Be Prepared!

Before you get started, take this readiness quiz.

1. Evaluate $5x - 4$ when $x = -1$.
   If you missed this problem, review Example 1.6.
2. Evaluate $3x - 2y$ when $x = 4, y = -3$.
   If you missed this problem, review Example 1.21.

If you missed this problem, review Example 2.2.

**Plot Points on a Rectangular Coordinate System**

Just like maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a rectangular coordinate system. The rectangular coordinate system is also called the $xy$-plane or the “coordinate plane.”

The rectangular coordinate system is formed by two intersecting number lines, one horizontal and one vertical. The horizontal number line is called the $x$-axis. The vertical number line is called the $y$-axis. These axes divide a plane into four regions, called quadrants. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise. See Figure 3.2.

In the rectangular coordinate system, every point is represented by an **ordered pair**. The first number in the ordered pair is the $x$-coordinate of the point, and the second number is the $y$-coordinate of the point. The phrase “ordered pair” means that the order is important.

**Ordered Pair**

An **ordered pair**, $(x, y)$ gives the coordinates of a point in a rectangular coordinate system. The first number is the $x$-coordinate. The second number is the $y$-coordinate.

What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the **origin**.

**The Origin**

The point $(0, 0)$ is called the **origin**. It is the point where the $x$-axis and $y$-axis intersect.

We use the coordinates to locate a point on the $xy$-plane. Let’s plot the point $(1, 3)$ as an example. First, locate 1 on the $x$-axis and lightly sketch a vertical line through $x = 1$. Then, locate 3 on the $y$-axis and sketch a horizontal line through $y = 3$. Now, find the point where these two lines meet—that is the point with coordinates $(1, 3)$. See Figure 3.3.
Notice that the vertical line through \( x = 1 \) and the horizontal line through \( y = 3 \) are not part of the graph. We just used them to help us locate the point \((1, 3)\).

When one of the coordinate is zero, the point lies on one of the axes. In Figure 3.4 the point \((0, 4)\) is on the \(y\)-axis and the point \((-2, 0)\) is on the \(x\)-axis.

**Points on the Axes**

Points with a \(y\)-coordinate equal to 0 are on the \(x\)-axis, and have coordinates \((a, 0)\).

Points with an \(x\)-coordinate equal to 0 are on the \(y\)-axis, and have coordinates \((0, b)\).

**EXAMPLE 3.1**

Plot each point in the rectangular coordinate system and identify the quadrant in which the point is located:

- \((-5, 4)\)
- \((-3, -4)\)
- \((2, -3)\)
- \((0, -1)\)
- \((3, \frac{5}{2})\)

**Solution**

The first number of the coordinate pair is the \(x\)-coordinate, and the second number is the \(y\)-coordinate. To plot each point, sketch a vertical line through the \(x\)-coordinate and a horizontal line through the \(y\)-coordinate. Their intersection is the point.

1. Since \( x = -5 \), the point is to the left of the \(y\)-axis. Also, since \( y = 4 \), the point is above the \(x\)-axis. The point \((-5, 4)\) is in Quadrant II.

2. Since \( x = -3 \), the point is to the left of the \(y\)-axis. Also, since \( y = -4 \), the point is below the \(x\)-axis. The point
(-3, -4) is in Quadrant III.

○ Since \( x = 2 \), the point is to the right of the \( y \)-axis. Since \( y = -3 \), the point is below the \( x \)-axis. The point \((2, -3)\) is in Quadrant IV.

○ Since \( x = 0 \), the point whose coordinates are \((0, -1)\) is on the \( y \)-axis.

○ Since \( x = 3 \), the point is to the right of the \( y \)-axis. Since \( y = \frac{5}{2} \), the point is above the \( x \)-axis. (It may be helpful to write \( \frac{5}{2} \) as a mixed number or decimal.) The point \(\left(3, \frac{5}{2}\right)\) is in Quadrant I.

\[ \text{\begin{tabular}{l}
\hline
\textbf{Quadrants} \\
\hline
\textbf{Quadrant I} & \textbf{Quadrant II} & \textbf{Quadrant III} & \textbf{Quadrant IV} \\
\hline
(x, y) & (x, y) & (x, y) & (x, y) \\
(+, +) & (-, +) & (-, -) & (+, -) \\
\hline
\end{tabular}} \]

\[ \text{This OpenStax book is available for free at http://cnx.org/content/col12119/1.5} \]
Up to now, all the equations you have solved were equations with just one variable. In almost every case, when you solved the equation you got exactly one solution. But equations can have more than one variable. Equations with two variables may be of the form $Ax + By = C$. An equation of this form is called a **linear equation** in two variables.

**Linear Equation**

An equation of the form $Ax + By = C$, where $A$ and $B$ are not both zero, is called a **linear equation** in two variables.

Here is an example of a linear equation in two variables, $x$ and $y$.

$Ax + By = C$

$x + 4y = 8$

$A = 1, B = 4, C = 8$

The equation $y = -3x + 5$ is also a linear equation. But it does not appear to be in the form $Ax + By = C$. We can use the Addition Property of Equality and rewrite it in $Ax + By = C$ form.

$$y = -3x + 5$$

Add to both sides.

$$y + 3x = -3x + 5 + 3x$$

Simplify.

$$y + 3x = 5$$

Use the Commutative Property to put it in $Ax + By = C$ form.

$$3x + y = 5$$

By rewriting $y = -3x + 5$ as $3x + y = 5$, we can easily see that it is a linear equation in two variables because it is of the form $Ax + By = C$. When an equation is in the form $Ax + By = C$, we say it is in **standard form of a linear equation**.

**Standard Form of Linear Equation**

A linear equation is in **standard form** when it is written $Ax + By = C$.

Most people prefer to have $A$, $B$, and $C$ be integers and $A \geq 0$ when writing a linear equation in standard form, although it is not strictly necessary.

Linear equations have infinitely many solutions. For every number that is substituted for $x$ there is a corresponding $y$ value. This pair of values is a **solution** to the linear equation and is represented by the ordered pair $(x, y)$. When we substitute these values of $x$ and $y$ into the equation, the result is a true statement, because the value on the left side is equal to the value on the right side.

**Solution of a Linear Equation in Two Variables**

An ordered pair $(x, y)$ is a **solution** of the linear equation $Ax + By = C$, if the equation is a true statement when the $x$- and $y$-values of the ordered pair are substituted into the equation.
Linear equations have infinitely many solutions. We can plot these solutions in the rectangular coordinate system. The points will line up perfectly in a straight line. We connect the points with a straight line to get the graph of the equation. We put arrows on the ends of each side of the line to indicate that the line continues in both directions.

A graph is a visual representation of all the solutions of the equation. It is an example of the saying, “A picture is worth a thousand words.” The line shows you all the solutions to that equation. Every point on the line is a solution of the equation. And, every solution of this equation is on this line. This line is called the graph of the equation. Points not on the line are not solutions!

**Graph of a Linear Equation**

The graph of a linear equation $Ax + By = C$ is a straight line.

- Every point on the line is a solution of the equation.
- Every solution of this equation is a point on this line.

**EXAMPLE 3.2**

The graph of $y = 2x - 3$ is shown.

For each ordered pair, decide:

ⓐ Is the ordered pair a solution to the equation?

ⓑ Is the point on the line?

A: $(0, -3)$  B: $(3, 3)$  C: $(2, -3)$  D: $(-1, -5)$

**Solution**

Substitute the $x$- and $y$-values into the equation to check if the ordered pair is a solution to the equation.

ⓐ

- $A$: $(0, -3)$
  
  $y = 2x - 3$
  
  $y = 2(0) - 3$
  
  $y = -3$ ✓
  
  $(0, -3)$ is a solution.

- $B$: $(3, 3)$
  
  $y = 2x - 3$
  
  $y = 2(3) - 3$
  
  $y = 3$ ✓
  
  $(3, 3)$ is a solution.

- $C$: $(2, -3)$
  
  $y = 2x - 3$
  
  $y = 2(2) - 3$
  
  $y = -3$ ✓
  
  $(2, -3)$ is not a solution.

- $D$: $(-1, -5)$
  
  $y = 2x - 3$
  
  $y = 2(-1) - 3$
  
  $y = -5$ ✓
  
  $(-1, -5)$ is a solution.

ⓑ Plot the points $(0, -3)$, $(3, 3)$, $(2, -3)$, and $(-1, -5)$.
The points \((0, 3), (3, -3),\) and \((-1, -5)\) are on the line \(y = 2x - 3,\) and the point \((2, -3)\) is not on the line.

The points that are solutions to \(y = 2x - 3\) are on the line, but the point that is not a solution is not on the line.

TRY IT : 3.3
Use graph of \(y = 3x - 1.\) For each ordered pair, decide:

-(a) Is the ordered pair a solution to the equation?
- (b) Is the point on the line?

\(A\) \((0, -1)\) \(B\) \((2, 5)\)
TRY IT : : 3.4 Use graph of $y = 3x - 1$. For each ordered pair, decide:

ⓐ Is the ordered pair a solution to the equation?
ⓑ Is the point on the line?

A $(3, -1)$ B $(-1, -4)$

Graph a Linear Equation by Plotting Points

There are several methods that can be used to graph a linear equation. The first method we will use is called plotting points, or the Point-Plotting Method. We find three points whose coordinates are solutions to the equation and then plot them in a rectangular coordinate system. By connecting these points in a line, we have the graph of the linear equation.

EXAMPLE 3.3 HOW TO GRAPH A LINEAR EQUATION BY PLOTTING POINTS

Graph the equation $y = 2x + 1$ by plotting points.
Solution

**Step 1.** Find three points whose coordinates are solutions to the equation.

You can choose any values for \(x\) or \(y\).

In this case, since \(y\) is isolated on the left side of the equation, it is easier to choose values for \(x\).

\[
\begin{align*}
\text{For } x = 0, & \quad y = 2 \cdot 0 + 1 = 1 \\
\text{For } x = 1, & \quad y = 2 \cdot 1 + 1 = 3 \\
\text{For } x = -2, & \quad y = 2 \cdot (-2) + 1 = -3
\end{align*}
\]

Organize the solutions in a table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>(0, 1)</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>(1, 3)</td>
</tr>
<tr>
<td>-2</td>
<td>-3</td>
<td>(-2, -3)</td>
</tr>
</tbody>
</table>

**Step 2.** Plot the points in a rectangular coordinate system.

Check that the points line up. If they do not, carefully check your work!

Plot: \((0, 1), (1, 3), (-2, -3)\).

Do the points line up? Yes, the points line up.

![Graph showing the plotted points](image-url)
Graph the equation by plotting points:

\[ y = 2x - 3. \]

Graph the equation by plotting points:

\[ y = -2x + 4. \]

The steps to take when graphing a linear equation by plotting points are summarized here.

**HOW TO:** GRAPH A LINEAR EQUATION BY PLOTTING POINTS.

1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work.
3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

It is true that it only takes two points to determine a line, but it is a good habit to use three points. If you only plot two points and one of them is incorrect, you can still draw a line but it will not represent the solutions to the equation. It will be the wrong line.

If you use three points, and one is incorrect, the points will not line up. This tells you something is wrong and you need to check your work. Look at the difference between these illustrations.

![Illustrations](image)

When an equation includes a fraction as the coefficient of \( x \), we can still substitute any numbers for \( x \). But the arithmetic is easier if we make “good” choices for the values of \( x \). This way we will avoid fractional answers, which are hard to graph precisely.

**EXAMPLE 3.4**

Graph the equation: \( y = \frac{1}{2}x + 3. \)

**Solution**

Find three points that are solutions to the equation. Since this equation has the fraction \( \frac{1}{2} \) as a coefficient of \( x \), we will
choose values of \( x \) carefully. We will use zero as one choice and multiples of 2 for the other choices. Why are multiples of two a good choice for values of \( x \)? By choosing multiples of 2 the multiplication by \( \frac{1}{2} \) simplifies to a whole number.

\[
\begin{array}{c|c|c|c}
 x & 0 & 2 & 4 \\
 y & \frac{1}{2}x + 3 & \frac{1}{2}x + 3 & \frac{1}{2}x + 3 \\
 y & \frac{1}{2}(0) + 3 & \frac{1}{2}(2) + 3 & \frac{1}{2}(4) + 3 \\
 y & 0 + 3 & 1 + 3 & 2 + 3 \\
 y & 3 & 4 & 5 \\
\end{array}
\]

The points are shown in Table 3.1.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>(2, 4)</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>(4, 5)</td>
</tr>
</tbody>
</table>

Table 3.1

Plot the points, check that they line up, and draw the line.

\[ y = \frac{1}{2}x + 3 \]

Graph Vertical and Horizontal Lines

Some linear equations have only one variable. They may have just \( x \) and no \( y \), or just \( y \) without an \( x \). This changes how we make a table of values to get the points to plot.

Let’s consider the equation \( x = -3 \). This equation has only one variable, \( x \). The equation says that \( x \) is always equal to
−3, so its value does not depend on y. No matter what is the value of y, the value of x is always −3.

So to make a table of values, write −3 in for all the x-values. Then choose any values for y. Since x does not depend on y, you can choose any numbers you like. But to fit the points on our coordinate graph, we’ll use 1, 2, and 3 for the y-coordinates. See Table 3.2.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−3</td>
<td>1</td>
<td>(−3, 1)</td>
</tr>
<tr>
<td>−3</td>
<td>2</td>
<td>(−3, 2)</td>
</tr>
<tr>
<td>−3</td>
<td>3</td>
<td>(−3, 3)</td>
</tr>
</tbody>
</table>

**Table 3.2**

Plot the points from the table and connect them with a straight line. Notice that we have graphed a **vertical line**.

![Graph of vertical line](image)

What if the equation has y but no x? Let’s graph the equation \( y = 4 \). This time the y-value is a constant, so in this equation, y does not depend on x. Fill in 4 for all the y’s in Table 3.3 and then choose any values for x. We’ll use 0, 2, and 4 for the x-coordinates.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>(0, 4)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>(2, 4)</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>(4, 4)</td>
</tr>
</tbody>
</table>

**Table 3.3**

In this figure, we have graphed a **horizontal line** passing through the y-axis at 4.
Vertical and Horizontal Lines

A **vertical line** is the graph of an equation of the form \( x = a \).

The line passes through the \( x \)-axis at \((a, 0)\).

A **horizontal line** is the graph of an equation of the form \( y = b \).

The line passes through the \( y \)-axis at \((0, b)\).

**EXAMPLE 3.5**

Graph: \( \circlearrowleft \) \( x = 2 \) \( \circlearrowright \) \( y = -1 \).

**Solution**

The equation has only one variable, \( x \), and \( x \) is always equal to 2. We create a table where \( x \) is always 2 and then put in any values for \( y \). The graph is a vertical line passing through the \( x \)-axis at 2.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>(2, 1)</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>(2, 2)</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>(2, 3)</td>
</tr>
</tbody>
</table>
Similarly, the equation \( y = -1 \) has only one variable, \( y \). The value of \( y \) is constant. All the ordered pairs in the next table have the same \( y \)-coordinate. The graph is a horizontal line passing through the \( y \)-axis at \(-1\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>(0, -1)</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>(3, -1)</td>
</tr>
<tr>
<td>-3</td>
<td>-1</td>
<td>(-3, -1)</td>
</tr>
</tbody>
</table>

Try It : : 3.9
Graph the equations: \( x = 5 \) \( y = -4 \).

Try It : : 3.10
Graph the equations: \( x = -2 \) \( y = 3 \).

What is the difference between the equations \( y = 4x \) and \( y = 4 \)?

The equation \( y = 4x \) has both \( x \) and \( y \). The value of \( y \) depends on the value of \( x \), so the \( y \)-coordinate changes according
to the value of \( x \). The equation \( y = 4 \) has only one variable. The value of \( y \) is constant, it does not depend on the value of \( x \), so the \( y \)-coordinate is always 4.

\[
\begin{array}{|c|c|c|}
\hline
\text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } \\
\hline
\end{array}
\]

Notice, in the graph, the equation \( y = 4x \) gives a slanted line, while \( y = 4 \) gives a horizontal line.

**EXAMPLE 3.6**

Graph \( y = -3x \) and \( y = -3 \) in the same rectangular coordinate system.

**Solution**

We notice that the first equation has the variable \( x \), while the second does not. We make a table of points for each equation and then graph the lines. The two graphs are shown.
Graph the equations in the same rectangular coordinate system: \( y = -4x \) and \( y = -4 \).

Graph the equations in the same rectangular coordinate system: \( y = 3 \) and \( y = 3x \).

**Find \( x \)- and \( y \)-intercepts**

Every linear equation can be represented by a unique line that shows all the solutions of the equation. We have seen that when graphing a line by plotting points, you can use any three solutions to graph. This means that two people graphing the line might use different sets of three points.

At first glance, their two lines might not appear to be the same, since they would have different points labeled. But if all the work was done correctly, the lines should be exactly the same. One way to recognize that they are indeed the same line is to look at where the line crosses the \( x \)-axis and the \( y \)-axis. These points are called the **intercepts of a line**.

**Intercepts of a Line**

The points where a line crosses the \( x \)-axis and the \( y \)-axis are called the **intercepts of the line**.

Let’s look at the graphs of the lines.

(a) \( 2x + y = 6 \)

(b) \( 3x - 4y = 12 \)

(c) \( x - y = 5 \)

(d) \( y = -2x \)

First, notice where each of these lines crosses the \( x \)-axis. See Table 3.4.

Now, let’s look at the points where these lines cross the \( y \)-axis.
Table 3.4

Do you see a pattern?

For each line, the $y$-coordinate of the point where the line crosses the $x$-axis is zero. The point where the line crosses the $x$-axis has the form $(a, 0)$ and is called the $x$-intercept of the line. The $x$-intercept occurs when $y$ is zero.

In each line, the $x$-coordinate of the point where the line crosses the $y$-axis is zero. The point where the line crosses the $y$-axis has the form $(0, b)$ and is called the $y$-intercept of the line. The $y$-intercept occurs when $x$ is zero.

<table>
<thead>
<tr>
<th>Figure</th>
<th>The line crosses the $x$-axis at:</th>
<th>Ordered pair for this point</th>
<th>The line crosses the $y$-axis at:</th>
<th>Ordered pair for this point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure (a)</td>
<td>3</td>
<td>$(3, 0)$</td>
<td>6</td>
<td>$(0, 6)$</td>
</tr>
<tr>
<td>Figure (b)</td>
<td>4</td>
<td>$(4, 0)$</td>
<td>$-3$</td>
<td>$(0, -3)$</td>
</tr>
<tr>
<td>Figure (c)</td>
<td>5</td>
<td>$(5, 0)$</td>
<td>$-5$</td>
<td>$(0, 5)$</td>
</tr>
<tr>
<td>Figure (d)</td>
<td>0</td>
<td>$(0, 0)$</td>
<td>0</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>General Figure</td>
<td>$a$</td>
<td>$(a, 0)$</td>
<td>$b$</td>
<td>$(0, b)$</td>
</tr>
</tbody>
</table>

**x-intercept and y-intercept of a Line**

The $x$-intercept is the point $(a, 0)$ where the line crosses the $x$-axis.

The $y$-intercept is the point $(0, b)$ where the line crosses the $y$-axis.

- The $x$-intercept occurs when $y$ is zero.
- The $y$-intercept occurs when $x$ is zero.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$b$</td>
</tr>
</tbody>
</table>

**EXAMPLE 3.7**

Find the $x$- and $y$-intercepts on each graph shown.
Solution

(a) The graph crosses the $x$-axis at the point $(4, 0)$. The $x$-intercept is $(4, 0)$.
The graph crosses the $y$-axis at the point $(0, 2)$. The $y$-intercept is $(0, 2)$.

(b) The graph crosses the $x$-axis at the point $(2, 0)$. The $x$-intercept is $(2, 0)$.
The graph crosses the $y$-axis at the point $(0, −6)$. The $y$-intercept is $(0, −6)$.

(c) The graph crosses the $x$-axis at the point $(-5, 0)$. The $x$-intercept is $(-5, 0)$.
The graph crosses the $y$-axis at the point $(0, −5)$. The $y$-intercept is $(0, −5)$. 
Recognizing that the $x$-intercept occurs when $y$ is zero and that the $y$-intercept occurs when $x$ is zero, gives us a method to find the intercepts of a line from its equation. To find the $x$-intercept, let $y = 0$ and solve for $x$. To find the $y$-intercept, let $x = 0$ and solve for $y$.

### Find the $x$- and $y$-intercepts from the Equation of a Line

Use the equation of the line. To find:
- the $x$-intercept of the line, let $y = 0$ and solve for $x$.
- the $y$-intercept of the line, let $x = 0$ and solve for $y$.

**EXAMPLE 3.8**

Find the intercepts of $2x + y = 8$.

**Solution**

We will let $y = 0$ to find the $x$-intercept, and let $x = 0$ to find the $y$-intercept. We will fill in a table, which reminds us of what we need to find.

| $2x + y = 8$ |  
|---|---|
| $x$ | $y$ |
| 0 | 0 | $x$-intercept |
| 0 | 8 | $y$-intercept |
To find the $x$-intercept, let $y = 0$.

\[
\begin{align*}
2x + y &= 8 \\
\text{Let } y &= 0. \\
2x + 0 &= 8 \\
\text{Simplify.} \\
2x &= 8 \\
\text{ } &= 4 \\
\text{The } x\text{-intercept is:} \\
\text{ } &\quad (4, \ 0)
\end{align*}
\]

To find the $y$-intercept, let $x = 0$.

\[
\begin{align*}
2x + y &= 8 \\
\text{Let } x &= 0. \\
2 \cdot 0 + y &= 8 \\
\text{Simplify.} \\
0 + y &= 8 \\
\text{ } &= 8 \\
\text{The } y\text{-intercept is:} \\
\text{ } &\quad (0, \ 8)
\end{align*}
\]

The intercepts are the points $(4, \ 0)$ and $(0, \ 8)$ as shown in the table.

| $2x + y = 8$ |
|---|---|
| $x$ | $y$ |
| 4 | 0 |
| 0 | 8 |

**TRY IT : : 3.15** Find the intercepts: $3x + y = 12$.

**TRY IT : : 3.16** Find the intercepts: $x + 4y = 8$.

**Graph a Line Using the Intercepts**

To graph a linear equation by plotting points, you need to find three points whose coordinates are solutions to the equation. You can use the $x$- and $y$-intercepts as two of your three points. Find the intercepts, and then find a third point to ensure accuracy. Make sure the points line up—then draw the line. This method is often the quickest way to graph a line.

**EXAMPLE 3.9** HOW TO GRAPH A LINE USING THE INTERCEPTS

Graph $-x + 2y = 6$ using the intercepts.
Solution

Step 1. Find the x- and y-intercepts of the line.
Let \( y = 0 \) and solve for \( x \).

Let \( x = 0 \) and solve for \( y \).

Find the x-intercept.

Find the y-intercept.

Let \( y = 0 \)

\(-x + 2y = 6\)

\(-x + 2(0) = 6\)

\(-x = 6\)

\(x = -6\)

The x-intercept is \((-6, 0)\).

Let \( x = 0 \).

\(-x + 2y = 6\)

\(-0 + 2y = 6\)

\(2y = 6\)

\(y = 3\)

The y-intercept is \((0, 3)\).

Step 2. Find another solution to the equation.
We’ll use \( x = 2 \).

Let \( x = 2 \).

\(-x + 2y = 6\)

\(-2 + 2y = 6\)

\(2y = 8\)

\(y = 4\)

A third point is \((2, 4)\).

Step 3. Plot the three points. Check that the points line up.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>0</td>
<td>(-6, 0)</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>(2, 4)</td>
</tr>
</tbody>
</table>

Step 4. Draw the line.
See the graph.

TRY IT 3.17 Graph using the intercepts: \( x - 2y = 4 \).
The steps to graph a linear equation using the intercepts are summarized here.

**HOW TO :: GRAPH A LINEAR EQUATION USING THE INTERCEPTS.**

1. Find the x- and y-intercepts of the line.
   - Let \( y = 0 \) and solve for \( x \).
   - Let \( x = 0 \) and solve for \( y \).
2. Find a third solution to the equation.
3. Plot the three points and check that they line up.
4. Draw the line.

**EXAMPLE 3.10**

Graph \( 4x - 3y = 12 \) using the intercepts.

**Solution**

Find the intercepts and a third point.

<table>
<thead>
<tr>
<th>x-intercept, let ( y = 0 )</th>
<th>y-intercept, let ( x = 0 )</th>
<th>third point, let ( y = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 4x - 3y = 12 )</td>
<td>( 4x - 3y = 12 )</td>
<td>( 4x - 3y = 12 )</td>
</tr>
<tr>
<td>( 4x - 3(0) = 12 )</td>
<td>( 4(0) - 3y = 12 )</td>
<td>( 4x - 3(4) = 12 )</td>
</tr>
<tr>
<td>( 4x = 12 )</td>
<td>( -3y = 12 )</td>
<td>( 4x - 12 = 12 )</td>
</tr>
<tr>
<td>( x = 3 )</td>
<td>( y = -4 )</td>
<td>( 4x = 24 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x = 6 )</td>
</tr>
</tbody>
</table>

We list the points in the table and show the graph.

<table>
<thead>
<tr>
<th>( 4x - 3y = 12 )</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>
Graph using the intercepts: $5x - 2y = 10$.

Graph using the intercepts: $3x - 4y = 12$.

When the line passes through the origin, the x-intercept and the y-intercept are the same point.
EXAMPLE 3.11

Graph $y = 5x$ using the intercepts.

Solution

This line has only one intercept. It is the point $(0, 0)$.

To ensure accuracy, we need to plot three points. Since the $x$- and $y$-intercepts are the same point, we need two more points to graph the line.

The resulting three points are summarized in the table.

<table>
<thead>
<tr>
<th>$y = 5x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>-1</td>
</tr>
</tbody>
</table>

Plot the three points, check that they line up, and draw the line.
Graph the intercepts: \( y = -x \).
3.1 EXERCISES

Practice Makes Perfect

Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system and identify the quadrant in which the point is located.

1. ⪯ (−4, 2) ⪯ (−1, −2) ⪯ (3, −5) ⪯ (−3, 0) ⪯ \( \left( \frac{5}{3}, 2 \right) \)

2. ⪯ (−2, −3) ⪯ (3, −3) ⪯ (−4, 1) ⪯ (4, −1) ⪯ \( \left( \frac{3}{2}, 1 \right) \)

3. ⪯ (3, −1) ⪯ (−3, 1) ⪯ (−2, 0) ⪯ (−4, −3) ⪯ \( \left( 1, \frac{14}{5} \right) \)

4. ⪯ (−1, 1) ⪯ (−2, −1) ⪯ (2, 0) ⪯ (1, −4) ⪯ \( \left( 3, \frac{7}{2} \right) \)

In the following exercises, for each ordered pair, decide

○ is the ordered pair a solution to the equation? ○ is the point on the line?

5. \( y = x + 2 \);
   A: (0, 2); B: (1, 2); C: (−1, 1); D: (−3, −1).

6. \( y = x − 4 \);
   A: (0, −4); B: (3, −1); C: (2, 2); D: (1, −5).
7. \( y = \frac{1}{2}x - 3; \)  
   A: (0, -3); B: (2, -2); C: (-2, -4); D: (4, 1)  

8. \( y = \frac{1}{3}x + 2; \)  
   A: (0, 2); B: (3, 3); C: (-3, 2); D: (-6, 0).

---

**Graph a Linear Equation by Plotting Points**  
*In the following exercises, graph by plotting points.*

9. \( y = x + 2 \)  
10. \( y = x - 3 \)  
11. \( y = 3x - 1 \)  
12. \( y = -2x + 2 \)  
13. \( y = -x - 3 \)  
14. \( y = -x - 2 \)  
15. \( y = 2x \)  
16. \( y = -2x \)  
17. \( y = \frac{1}{2}x + 2 \)  
18. \( y = \frac{1}{3}x - 1 \)  
19. \( y = \frac{4}{3}x - 5 \)  
20. \( y = \frac{3}{2}x - 3 \)  
21. \( y = -\frac{2}{5}x + 1 \)  
22. \( y = -\frac{4}{5}x - 1 \)  
23. \( y = -\frac{3}{2}x + 2 \)  
24. \( y = -\frac{5}{3}x + 4 \)

---

**Graph Vertical and Horizontal lines**  
*In the following exercises, graph each equation.*

25. (a) \( x = 4 \) (b) \( y = 3 \)  
26. (a) \( x = 3 \) (b) \( y = 1 \)  
27. (a) \( x = -2 \) (b) \( y = -5 \)  
28. (a) \( x = -5 \) (b) \( y = -2 \)

---

*In the following exercises, graph each pair of equations in the same rectangular coordinate system.*

29. \( y = 2x \) and \( y = 2 \)  
30. \( y = 5x \) and \( y = 5 \)  
31. \( y = -\frac{1}{2}x \) and \( y = -\frac{1}{2} \)  
32. \( y = -\frac{1}{3}x \) and \( y = -\frac{1}{3} \)
Find x- and y-Intercepts

In the following exercises, find the x- and y-intercepts on each graph.

33. 

34. 

35. 

36. 

In the following exercises, find the intercepts for each equation.

37. \( x - y = 5 \)  
38. \( x - y = -4 \)  
39. \( 3x + y = 6 \)  
40. \( x - 2y = 8 \)  
41. \( 4x - y = 8 \)  
42. \( 5x - y = 5 \)  
43. \( 2x + 5y = 10 \)  
44. \( 3x - 2y = 12 \)  

Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

45. \( -x + 4y = 8 \)  
46. \( x + 2y = 4 \)  
47. \( x + y = -3 \)  
48. \( x - y = -4 \)  
49. \( 4x + y = 4 \)  
50. \( 3x + y = 3 \)  
51. \( 3x - y = -6 \)  
52. \( 2x - y = -8 \)  
53. \( 2x + 4y = 12 \)  
54. \( 3x - 2y = 6 \)  
55. \( 2x - 5y = -20 \)  
56. \( 3x - 4y = -12 \)  
57. \( y = -2x \)  
58. \( y = 5x \)  
59. \( y = x \)
60. \( y = -x \)

**Mixed Practice**

*In the following exercises, graph each equation.*

61. \( y = \frac{3}{2}x \)
62. \( y = -\frac{2}{3}x \)
63. \( y = -\frac{1}{2}x + 3 \)
64. \( y = \frac{1}{4}x - 2 \)
65. \( 4x + y = 2 \)
66. \( 5x + 2y = 10 \)
67. \( y = -1 \)
68. \( x = 3 \)

**Writing Exercises**

69. Explain how you would choose three \( x \)-values to make a table to graph the line \( y = \frac{1}{5}x - 2 \).

70. What is the difference between the equations of a vertical and a horizontal line?

71. Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation \( 4x + y = -4 \)? Why?

72. Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation \( y = \frac{2}{3}x - 2 \)? Why?

**Self Check**

After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

<table>
<thead>
<tr>
<th>I can...</th>
<th>Confidently</th>
<th>With some help</th>
<th>No—I don’t get it!</th>
</tr>
</thead>
<tbody>
<tr>
<td>plot points on a rectangular coordinate system.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>graph a linear equation by plotting points.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>graph vertical and horizontal lines.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>find ( x )- and ( y )-intercepts.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>graph a line using the intercepts.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If most of your checks were:

Confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

With some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

No, I don’t get it. This is a warning sign and you must address it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.
3.2  Slope of a Line

Learning Objectives

By the end of this section, you will be able to:

- Find the slope of a line
- Graph a line given a point and the slope
- Graph a line using its slope and intercept
- Choose the most convenient method to graph a line
- Graph and interpret applications of slope–intercept
- Use slopes to identify parallel and perpendicular lines

Be Prepared!

Before you get started, take this readiness quiz.

1. Simplify: \((1 - 4) \div (8 - 2)\).
   
   If you missed this problem, review Example 1.30.

2. Divide: \(0 \div 4\).
   
   If you missed this problem, review Example 1.49.

3. Simplify: \(\frac{15}{3}, \frac{-15}{3}, \frac{-15}{3}\).
   
   If you missed this problem, review Example 1.30.

Find the Slope of a Line

When you graph linear equations, you may notice that some lines tilt up as they go from left to right and some lines tilt down. Some lines are very steep and some lines are flatter.

In mathematics, the measure of the steepness of a line is called the slope of the line. The concept of slope has many applications in the real world. In construction the pitch of a roof, the slant of the plumbing pipes, and the steepness of the stairs are all applications of slope. and as you ski or jog down a hill, you definitely experience slope.

We can assign a numerical value to the slope of a line by finding the ratio of the rise and run. The rise is the amount the vertical distance changes while the run measures the horizontal change, as shown in this illustration. Slope is a rate of change. See Figure 3.5.

\begin{figure}[h]
  \centering
  \includegraphics[width=0.3\textwidth]{slope_triangle.png}
  \caption{Slope of a Line}
\end{figure}

Slope of a Line

The slope of a line is \(m = \frac{\text{rise}}{\text{run}}\).

The rise measures the vertical change and the run measures the horizontal change.

To find the slope of a line, we locate two points on the line whose coordinates are integers. Then we sketch a right triangle where the two points are vertices and one side is horizontal and one side is vertical.

To find the slope of the line, we measure the distance along the vertical and horizontal sides of the triangle. The vertical distance is called the rise and the horizontal distance is called the run,